Measuring Antenna Gain Using the Friis 3-Antenna Method and a nanoVNA

AD6AE

"There are many methods of measuring the gain of an antenna; most of them call for a reference antenna of known gain.[1] *This method requires three antennas but does not require knowledge of any of them, but will find the gain of each of them. Since the method depends on the Friis transmission equation, all the conditions for its validity must apply." Owen Duffy VK2OMD*[2]

The Friis transmission equation was developed by Harald Friis *(pronounced Frisch)* in 1946. This article describes its application as related to the 3-antenna method of measuring antenna gain in a concise, straightforward manner.[3][16][20] However, careful attention to detail is critical for accuracy. The Gauss-Jordan Elimination equations used to calculate individual antenna gains were recomposed to enhance operability and comprehension then rigorously tested against standard calculators written by VK2OMD et al.[1][3][9]

Antenna range testing for the amateur is practical only for VHF/UHF. The ubiquitous *nanoVNA* has made it possible to measure antenna gain with relative ease and acceptable accuracy; measurement data, including compensation for distance and cable losses, are then plugged into a simplified elimination equation where the gain for each antenna is easily solved.

Range measurements are neither trivial nor exact but have several factors that improve accuracy include: properly calibrated equipment, range setup, correct antenna spacing and height; selecting a location that is unobstructed and free of adjacent buildings or metal structures that can cause multipathing; no overhead or close by power lines, and having enough open space to accommodate antenna heights of 3-4 meters and separations of 15 to 25 meters or more. When met, tests should yield results with acceptable levels of accuracy.[3][8][9][10]

In addition to gain, three other simple tests that should be concurrently performed on the *Antenna Being Evaluated* are: Beamwidth, Front to Back ratio, and Front to Side ratio.[4][5][6]

~~~~ Definition of Terms ~~~~

- **VNA Ports:** s11 is Rx on Port 1/Tx on Port 1 *(s11 REFL)*; s21 is Rx on Port 2/Tx on Port 1 *(s21 THRU)*.
- **AUT(s):** Antenna(s) Under Test.
- ***PL:*** Free Space Path Loss (FSPL) is signal level decrease as a function of distance and frequency.
- ***d:*** Distance between Rx & Tx antennas in km (scaled to allow entry in meters)[11][12] **(Notes 1.d)**.
- ***Lc*****:** Cable loss of the coax that connects the distant Rx AUT to VNA port s21.[6]
- ***32.44:*** A constant as a function of field intensity dispersion per unit of distance and used in computing PL.[13]
- ***f_0:*** Frequency of operation: center of span, sweep range or a stationary CW frequency.
- ***dBi*****:** Gain as referenced to an isotropic antenna having a spherical radiation pattern in free space.
- ***dBd:*** Gain as referenced to a dipole in free space.[17][19]
- **Conversions: Gain dBi** = dBd **+** 2.15 & **Gain dBd** = dBi **-** 2.15 [17]

~~~~ nanoVNA Calibration and Test Setup ~~~~

1. *Calibrate s11 REFL and s21 THRU* (**do not skip this step!**). Refresher: W2AEW video.[15]
   Open Menu, Tap Calibrate, Reset; Stimulus Start/Stop: 142MHz/150MHz; Perform O.S.L. cal. on port s11.
2. *Perform Isolation and Thru calibration* Move Load to port s21**;** Tap ISOLN; Remove Load; Connect a very short 4-6" coax cable from s11 to s21. Tap THRU, Done, Save (to a Channel).
3. *Assign a Trace to Display s21 as Logmag* Open Menu. Tap Display, tap "Channel s11 REFL" to toggle it to "s21 THRU". Tap Trace; Tap any trace tile to assign and display s21. Tap Format, Logmag, Back, Display, Trace; tap unused traces to toggle them to off; Save. This displays a single line showing the Rx signal in (-dB).
4. *Measure the Long Rx Cable for THRU Loss (Lc) and Calibrate s11 to an Extended Reference Plane (Tx)*
   Put Common Mode chokes on ***both*** Tx and Rx cables used for the test close to the connector at the antenna end. Connect the long cable that will be connected to the distant Rx antenna between ports 1 and 2 and measure *s21 THRU* loss; record as an unsigned number on the ***Lc*** line of the worksheet on **Page 4**.

5. ***Extend s11 Reference Plane*** Attach the Tx cable to *s11*; Open Menu; Tap Calibrate, Reset; Stimulus, Start/Stop freq. to 142MHz/150MHz; Perform O.S.L. calibration with standards placed at the far *end of the Tx cable* where the CM choke is attached, tap Save. The VNA's reference plane is now "extended" out to the antenna connector.

Equipment Used for Testing

- NanoVNA-H4 calibrated for s11 Extended and s21 THRU.
- 20-foot long (RG-316) cable, calibrated as the extended reference plane.
- 75-foot long (RG-316) cable, measured for THRU loss.
- The *Antenna Being Evaluated** plus two others (dipoles, Yagis or a mix); all tuned for a 1:1 VSWR at 146 MHz.
- Two stands with masts 3-4 meters high, having a 1-1.5 meter long PVC or fiberglass upper section.
- AUTs were chosen for being conceptually unique, and typical of old-school, ham radio "make do" spirit!
  - ➔ **Ant (1):** ½ wave Tape Measure Dipole
  - ➔ **Ant (2):** 2-element Tape Measure Yagi
  - ➔ **Ant (3):** 3-element Tape Measure Yagi *(antenna being evaluated)*

~~~~ Test Implementation ~~~~

1. To ensure far-field measurements,space AUTs 15-20 meters (≈ 50 to 66 feet) apart.[8]
2. For *PL* Use **Table 1** or, if using a different separation and/or frequency, recalculate PL **(Notes 1.e)**.
3. Horizontally orient, then elevate and boresight AUTs at a height of 3 to 4m (≈ 9 to 12 feet) **Fig. 1**.[7]
4. Connect the Tx AUT to s11 and the Rx AUT to s21. Test using the pairing order shown on the worksheet.
5. When finished testing *Pair 3-to-1*, leave undisturbedAUTs; test remaining parameters below.
6. **Beamwidth:** Record boresight heading of Ant 3; slowly turn it each side of the boresight until the Rx signal strength changes by *(-3 dB)*; record the compass headings; the difference should be 50° to 70° (**Fig 2)**.[4]
7. The VNA displays s21 levels in *(-dB)*. When measuring F/S levels, slightly swing the antenna L/R ≈30° stopping on the lobe with the least negative dB level. There may be several side lobes. Record only the least negative one. ***Note:*** *The highest Rx level attainable for antenna gain readings is (0 dB).*
8. **F/B Ratio:** Turn ant. 180°; boresight, record; Ratio=difference of F/B levels in dB.[5]
9. **F/S Ratio:** Turn back to 90°; jog L/R, record; Ratio=difference of F/S levels in dB.[6]
10. **Vertically Polarized Parameters**: Optional. Repeat all tests with the antennas vertically orientated.[7]

Notes 1:

a. s21 Rx readings on the VNA will display in *(-dB)*; record them as unsigned numbers on the worksheet.
b. When calculating gains, plug in values and use the signs shown in the equations.
c. For convenience and further study, create a spreadsheet similar to that of **Fig. 3**.
d. FSPL *(PL)* calc. is scaled so that distance *d* may be entered in meters *(dm)* instead of kilometers (dkm).
e. Excel equation for PL: =20*LOG(*dm* x 0.001) + 20*LOG(*fMHz*) + 32.44 [11][12][13] or use **Table 1**.

~ Precalculated Path Loss for Distance and Frequency ~

dm (ft)	(144 MHz)	(146 MHz)	(148 MHz)
10m (33')	35.6 dB	35.7 dB	35.9 dB
15m (50')	39.1 dB	39.3 dB	39.4 dB
20m (66')	41.6 dB	41.8 dB	41.9 dB
25m (82')	43.6 dB	43.7 dB	43.8 dB

~~~~~ Table 1 ~~~~~

References:

1. Inside Wireless: Gain Measurement - YouTube
2. O. Duffy, VK2OMD
3. 3 Point Antenna Measurements - Friis Method
4. Inside Wireless: Antenna Beam Width
5. Inside Wireless: Front to Back ratio
6. Inside Wireless: Side Lobes
7. Wave Polarization and Antenna Polarization
8. Near and far field - Wikipedia
9. How to Measure Antenna Gain in 5 Simple Steps
10. Antenna Theory - Reciprocity
11. Free Space Path Loss Calculator
12. Free Space Path Loss: Details & Calculator
13. fspl constant of 32.44
14. Measurement of Antenna Radiation Pattern
15. #379: How to measure coax loss using a NanoVNA.
16. Friis Equation - (aka Friis Transmission Formula)
17. dBi vs. dBd summary | Digi International
18. VHF/UHF Yagi Antenna Quick Designer - K7MEM
19. dBi vs. dBd (G9C04)
20. Friis transmission equation - Wikipedia

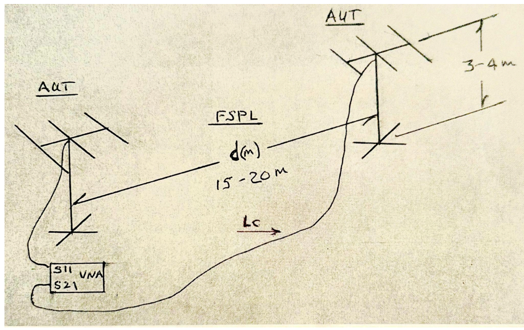

Fig. 1
Sketch of Range Setup

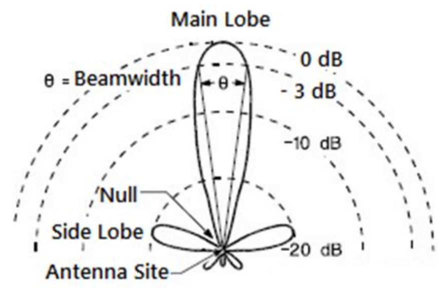

Fig. 2
Typical Yagi Antenna Radiation Pattern[14]
Beamwidth is the half-power or -3dB points

Notes 2:

1. Signs and computational order of the original elimination equations have been recomposed so that values can be entered as unsigned numbers. The revised format gives the equations a more "conventional look and feel" when manually performing calculations. The spreadsheet was similarly reformatted. **Fig. 3**.[3][20]
2. When utilizing the nanoVNA as shown in **Fig. 1**, s21 must be connected to the distant Rx AUT by a relatively long cable. Considering that cable loss (***Lc)*** is a function of both length and frequency, it must therefore be considered additional path loss. If not included, calculated gain results will be grossly in error.[12][15]
3. For convenience, *PL* is precalculated (**Table 1)** and illustrates the relationships between ***f*** and ***d***.

	A	B	C	D	E	F	G	H	I	J	K
1	**3-Antenna Gain Calculator**										
2		**Freq (MHz)**	146.0	**Make entries in yellow boxes only**			**Cell:**	**Calculation**			
3		***d*** **(meters)**	20.0	**Results**			E4	=(-C4-C6+C5+E7+C7)/2			
4		**s21 dB12 (Ant 1)**	46.7	**Antenna 1**	**0.92**	**(dBi)**	E5	=(-C5-C4+C6+E7+C7)/2			
5		**s21 dB23 (Ant 2)**	39.3	**Antenna 2**	**3.82**	**(dBi)**	E6	=(-C6-C5+C4+E7+C7)/2			
6		**s21 dB31 (Ant 3)**	42.2	**Antenna 3**	**8.32**	**(dBi)**	E7	=20*LOG(C2)+20*LOG(C3*0.001)+32.44			
7		**Lc** ***(cable loss)***	9.7	**PL** ***(path loss)***	**41.75**	**(dB)**	OR	=20*LOG(4*3.14159*C2*C3/299.792458)			
8											

Fig. 3
Screenshot of an Excel Path Loss and 3-Antenna Gain Calculator

Example of a Friis 3-Antenna Method Gain Worksheet

Record all dB values as unsigned

1-2 means Ant 1 is Tx and Ant 2 is Rx, etc.

Ant. 1-to-2 or $s21_{12}$ (A_1) **:** *46.7 dB*

Ant. 2-to-3 or $s21_{23}$ (A_2) **:** *39.3 dB*

Ant. 3-to-1 or $s21_{31}$ (A_3) **:** *42.2 dB*

Distance *(d)*: *20 m*

PL (from Table 1): *41.8 dB*

Conventional Elimination Gain Equation Order

Antenna 1: $(A_1) = (A_1 + A_3 - A_2 + PL + Lc) \div 2$

Antenna 2: $(A_2) = (A_2 + A_1 - A_3 + PL + Lc) \div 2$

Antenna 3: $(A_3) = (A_3 + A_2 - A_1 + PL + Lc) \div 2$

Frequency: *146 MHz*

Height: *3 m*

Lc: *9.7 dB*

Recomposed Elimination Gain Equation Order — Measured Levels

Ant (1) Gain $= (A_2 + P_L + L_C - A_1 - A_3) \div 2$ is $(39.3 + 41.8 + 9.7 - 46.7 - 42.2) \div 2$ = **Ant 1: 0.95 dBi**

Ant (2) Gain $= (A_3 + P_L + L_C - A_2 - A_1) \div 2$ is $(42.2 + 41.8 + 9.7 - 39.3 - 46.7) \div 2$ = **Ant 2: 3.85 dBi**

Ant (3) Gain $= (A_1 + P_L + L_C - A_3 - A_2) \div 2$ is $(46.7 + 41.8 + 9.7 - 42.2 - 39.3) \div 2$ = **Ant 3: 8.35 dBi***

Other Parameters: Beamwidth: 58° F/B Ratio: 13 dB F/S Ratio: 35 dB

**Antenna Being Evaluated* (Gain closely agrees with a modeled K7MEM antenna made with standard materials.)

For results in dBd, subtract 2.15 from dBi

Example **(continued)**

Beamwidth: *Boresight* = 120° *Left* -3dB = 91° *Right* -3dB = 149° 149°- 91° = **58°**
F/B Ratio: F: 42.2 dB, **B:** 55 dB 55 - 42 = **13 dB**
F/S Ratio: F: 42.2 dB, **S:** 77 dB 77 - 42 = **35 dB**

Summary: As of this writing, I have found no mention of the Friis or Three Antenna Gain Method in any literature relating to Amateur Radio other than a white paper written by VK2OMD that he shared with me where he derived the method from Friis' original transmission equation into three easily solved, simultaneous equations. The few online professional sites that mention the Friis method, use either a tightly controlled engineered range or a scaled model in an anechoic chamber using a calibrated reference antenna and laboratory-grade equipment.{1}{3}

Applying the "3-Antenna Method" and using a nanoVNA to measure gain parameters is easy and perfectly suited for range testing VHF and UHF antennas by the amateur. It's fun, educational and a great opportunity to learn more about the nanoVNA's capabilities and increase technical skills.

Building off his white paper, this article explains an otherwise complex undertaking in concise, easy to understand terms with examples of actual field tests performed on three unknown antennas made of scrap material to demonstrate a process that's unique, and not widely known outside of professional circles. Using new and inexpensive technology like the nanoVNA, puts this proven method within easy reach of the amateur experimenter.

If you're building a new antenna using design calculators like those posted by K7MEM and DL6WU or have purchased an antenna with no enclosed data sheet or snapped up a bargain at a hamfest or swap-meet, then correctly using this method will provide acceptably accurate performance data.

Note: High gain antennas are of limited use in extending the range in a non-Line-of-Sight (non-LOS) environment. In a non-LOS environment the obstructions contribute more losses to the system than the antennas are able to overcome. Also, the obstructions cause the signals to bounce and arrive at the antenna from different angles so it is desirable to have an antenna with a wide beam width and a lower gain.{17}

Amateurs are encouraged to learn to use a nanoVNA alongside their antenna analyzer and ***quantify*** the phrase, ***"It works real good."*** The nanoVNA's low cost, versatility and accuracy makes it an amazing piece of test gear that will serve you well. *Lastly, I owe a debt of gratitude to VK2OMD for introducing this method to me and providing expert advice along the way. This was an educational experience and a challenging writeup.* 73's.

"Make everything as simple as possible, but not simpler." **~ Albert Einstein**

Friis 3-Antenna Method Gain Worksheet

Date: ___ /___ /20___ Site Location: ____________________________

Test Frequency: _____ MHz Height: _____ m

Ant. 1-to-2: (A_1) _____ dB Distance *d* : _____ m

Ant. 2-to-3: (A_2) _____ dB *PL* : _____ dB

Ant. 3-to-1: (A_3) _____ dB* *Lc*: _____ dB

Ant (1) Gain = (A_2 + PL + LC - A_1 - A_3) ÷ 2 = **Ant 1: ______ dBi**

Ant (2) Gain = (A_3 + PL + LC - A_2 - A_1) ÷ 2 = **Ant 2: ______ dBi**

Ant (3) Gain* = (A_1 + PL + LC - A_3 - A_2) ÷ 2 = **Ant 3: ______ dBi**

Beamwidth: *Boresight*. ____° *Left* -3dB. ____° *Right* -3dB. ____° = ____- ____ = ____°

F/B Ratio: *F.* ____ *dB / B.* ____ *dB* = ____ - ____ = _____ **dB**

F/S Ratio: *F.* ____ *dB / S.* ____ *dB* = ____ - ____ = _____ **dB**

****Antenna Being Evaluated***

NOTES: